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A Precise Study on Solving the Multi Server Queuing Model through Lpp By Using Python Program

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Abstract: Queuing systems focuses on waiting lines, the typical examples are, at ticket booking counters, super bazaars and at bank services, at public transport places etc. many people are waiting to receive the services.To reduce these waiting lines, a largernumber of servers are required. Increasing the number of servers means it is expenditure to the service provider, so the waiting linesand the number of servers must be optimized. In order to getting theoptimal solution, the perception of Linear Programming problem plays a significant role. In this article, the alternative techniques are used to solve the multi-server queuing model, for that the total expected cost of the multi-server queuing model problem translated in to linear programing problem subjected to the constraints and solved the linear programing problem with Python program and obtained the required number of servers and estimated number of customers in the system. And also evaluated the mean arrival rate and the mean service rate of the multi-serverqueuing system.

Key words: Linear programming problem, expected waiting cost, expected service cost, Multi server Queuing model, Python programming.

1. Introduction:

The topic linear programming (LP) is sub partition of an Operations Research which is very huge subject and the widevariety of difficulties are answered.Queuing theoryaddress and interpret the concept waiting lines. Some real-world examples arewaiting lines in Banks, Supermarkets and Public transport, etc. The queues are caused because of the deficiency of efficiency of the amenity system. By refining the mechanism of the system efficiency, the customer's expectation can be reached but it is adding up more expensive to the service provider. So, increasing the number of servers is not a right solution because it increases the idle time of system. Optimization techniques only help to face this kind of situation.

Optimization technique plays an important role to select and obtain the decision variables and its values that are maximize or minimize the objective function. Many research articles and books [1, 2] are available such as Kalaipriyan et. al (2017) used whale optimization algorithm for solving the combined heat and power economic dispatch[3]. Rajakumar et. al (2017) were used the Grey wolf optimization algorithm for finding location of unknown nodes in localization problem[4]. Venkatesh and Polaiah (2019), were proposed genetic optimization algorithm is used for detection of cancer in CT images of lung[5].In order to maximize the profit of the bakery, Akpan and Iwok (2016) used the idea of the Simplex technique to utilize the uncooked material [6].Konstantine and Georgakakos (2012), used the LP problem to formulate the inequalities of the problem and optimization function and established the algorithm andcompletedinvestigation of the data [7].

Vasanta kumat et al (2018) [8], Hanumanta Rao et al (2018) [9, 10], Swathi et al (2018) [11]and Vijay Prasad et al (2014, 2018) [12, 13, 14] studied and generalized performance measures of the various queuing system model.Sateesh et al (2018) [15], analyze the customer behavior at retail stores.Rajyalakshmi et al (2017,2018) [16, 17], and Kumar D (2017), [18, 19, 20] studied and analyzed the customer behavior and utilization of resources by using various statistical tools.
In this paper alternative techniques applied to solve the multi-server queuing models. The expected total cost of the multi-server queuing model problem converted in to LPP and solved with the help of the python program.

2. Formulating the Multi-server queuing problem in to Linear programming problem:

The linear programing problems are demonstrated by an optimization function which is to be minimized or maximized, subject to the inequalities.
Normal form of the LPP is defined as follows:

Minimize (or) Maximize $X= \sum_{i=1}^{m} A_i \alpha_i$

Subject to the inequalities $\sum_{i=1}^{m} b_{ki}\alpha_i \leq or = or \geq c_k,$ where $k = 1, \ldots, n$

$\alpha_i \geq 0,$ Where $i = 1, \ldots, m$

The formulation of the M/M/S queuing model as a linear programming problem is as follows:
For the given service cost per each server (Cs) and waiting cost per each customer (Cw), let S be the required number of servers, L_s be the expected number of the consumers in the system, then the expected total cost of multi-server queuing system is well-defined as follows:

Total cost $= Cs\, S + Cw\, Ls$

The expected total cost is to be minimized and the optimized function of LPP is defined as follows:

$Minimum\ Z = C_s\, S + C_w\ Ls$

subject to the inequalities $S + L_s \geq b_1, \quad S \geq b_2, \quad L_s \geq b_3, \qquad S, Ls > 0$

Here b_1 is the summation of the minimum number of servers and the minimum number of the consumers in the system b_2 is the minimum number of the service systems and b_3 is the minimum number of the consumers in the system.

3. Justification of the model:

The linear programing problemconsideredfor the given service cost of each server (C_s) Rs 150/- and average waiting cost of each consumer (C_w) Rs 100, the foremost objective is to find S and Ls at which the optimized value of optimal function is obtained as follows:

$Minimum\ Z = 150\, S + 100\ L_s,$ subject to the constraints $S + L_s \geq 12, \ L_s \geq 8, \ S \geq 2,$ where $S, L_s > 0$ and solving this LPP by using python program.

4. Python program:

```
import pulp as p
# Create a linear programing Minimization problem
Lp_prob = p.LpProblem('Problem', p.LpMinimize)
x = p.LpVariable("S", lowerBound = 0)   # Define a decision variable S>= 0
y = p.LpVariable("Ls", lowerBound = 0)   # Define a decision variable Ls>= 0
# Optimization Function
Lp_prob += 150 * S + 100 * Ls
# Constraints:
Lp_prob +=   1 * S + 1 * Ls>= 12
Lp_prob +=  0 * S+ 1 * Ls>= 8
Lp_prob += 1* S + 0 * Ls >= 2
Lp_prob += S>= 0
```

```
Lp_prob += Ls>= 0
print(Lp_prob)
status = Lp_prob.solve()
print(p.LpStatus[status])
print(p.value(S), p.value(Ls), p.value(Lp_prob.objective)) #end of the program
```

4.1Program Output:

```
Problem:
MINIMIZE
150*S + 100*Ls + 0
SUBJECT TO
C1: S + Ls>= 12
C2: Ls>= 8
C3: S>= 2
C4: S>= 0
C5: Ls>= 0
VARIABLES
S Continuous
Ls Continuous
Optimal
(2.0, 10.0, 1300.0).
```

From the output of python program, the values are obtained as follows:
S = 2, L_s = 10 and Minimum Z = 1300
The performance measures of the multi-server queuing model as follows:

The expected number of consumers in the queue $L_q = \left[\frac{\lambda\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2}\right]P_0$

where $P_0 = \left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)\right]^{-1}$

$$L_q = \frac{\frac{\lambda\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2}}{\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)}$$

$$= \frac{\lambda\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\left(\frac{s\mu}{s\mu-\lambda}\right)\right]}$$

$$L_q = \frac{\lambda\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu(s\mu-\lambda)\left(\lambda/\mu\right)^s}$$

The expected waiting time of a consumers in the queue $W_q = \frac{L_q}{\lambda}$

$$W_q = \frac{\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu(s\mu-\lambda)\left(\lambda/\mu\right)^s}$$

The expected waiting time of a consumers in the queue $W_s = W_q + \frac{1}{\mu}$

$$W_s = \frac{\mu\left(\lambda/\mu\right)^s}{(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu(s\mu-\lambda)\left(\lambda/\mu\right)^s} + \frac{1}{\mu}$$

$$W_s = \frac{\mu^2\left(\lambda/\mu\right)^s + (s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu(s\mu-\lambda)\left(\lambda/\mu\right)^s}{\mu(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu^2(s\mu-\lambda)\left(\lambda/\mu\right)^s}$$

The expected number of consumers in the queue $L_s = \lambda\, W_s$

$$L_s = \frac{\lambda\mu^2\left(\lambda/\mu\right)^s + \lambda(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \lambda\mu(s\mu-\lambda)\left(\lambda/\mu\right)^s}{\mu(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu^2(s\mu-\lambda)\left(\lambda/\mu\right)^s}$$

$$L_s\left[\mu(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \mu^2(s\mu-\lambda)\left(\lambda/\mu\right)^s\right] - \left[\lambda\mu^2\left(\lambda/\mu\right)^s + \lambda(s-1)!\,(s\mu-\lambda)^2\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \lambda\mu(s\mu-\lambda)\left(\lambda/\mu\right)^s\right] = 0$$

$$(s\mu-\lambda)(L_s-\lambda)\left[(s-1)!\,(s\mu-\lambda)\mu^{s-1}\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \lambda^s\right] - \mu\,\lambda^{s+1} = 0$$

Assume the above equality is $\psi(\lambda,\mu) = 0$

Where $\psi(\lambda,\mu) = (s\mu-\lambda)(L_s-\lambda)\left[(s-1)!\,(s\mu-\lambda)\mu^{s-1}\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \lambda^s\right] - \mu\,\lambda^{s+1}$

One can treat μ as a free variable (>0) and solve the above algebraic equation to get ($s+1$) values of λ among which one choose λ (>0) such that $\frac{\lambda}{s\mu} \in (0,1)$. Independency of choosing μ gives the assurance of the existence of the λ.

5. Conclusion:

In this research paper, the multi-server queuing model formulate intolinear programming problem (LPP) and obtained the required number of servers and expected number of consumers in the system through LPP by using Python programand also derived a function in terms of mean arrival rate (λ) and mean service rate (μ) One can treat μ as a free variable (>0) and solve the equation to get ($s+1$) values of λ among which one choose λ (>0) such that $\frac{\lambda}{s\mu} \in (0,1)$. Independency of choosing μ gives the assurance of the existence of the λ.

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